# Physics I
# ISI B.Math
# Backpaper Exam : Jume 14 , 2018

Total Marks: 50

Answer all questions

1. (**Marks** $= \mathbf{6} + \mathbf{4} = \mathbf{10}$)

A damped oscillator satisfies the equation

$$\ddot{x} + 2K\dot{x} + \Omega^2 x = 0$$

where $K$ and $\Omega$ are positive constants with $K < \Omega$( underdamping). At time $t = 0$ the particle is released from rest at the point $x = a$.
(a) Show that the subsequent motion is given by

$$x = ae^{-Kt}\left(\cos \Omega_D t + \frac{K}{\Omega_D} \sin \Omega_D t\right)$$

where $\Omega_D = (\Omega^2 - K^2)^{\frac{1}{2}}$.

(b) Find all the turning points of the function $x(t)$ and show that the ratio of the successive maximum values of $x$ is $e^{\frac{-2\pi K}{\Omega_D}}$

2. (**Marks** $= \mathbf{4} + \mathbf{6} = \mathbf{10}$)

(a) A particle of mass $m$ moves in the central force field $\mathbf{F} = -(\frac{m\gamma}{r^2})\hat{\mathbf{r}}$ , where $\gamma$ is a positive constant. Show that bounded and unbounded orbits are possible depending on the value of the total energy $E$

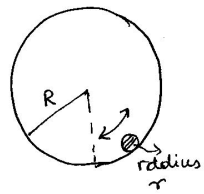
(b) An asteroid is approaching the sun from a great distance. At this time it has a constant speed $u$ and is moving in a straight line whose perpendicular distance from the sun is $p$. For the special case in which $u^2 = \frac{4M_S G}{3p}$ (where $M_S$ is the mass of the Sun, and $G$ the gravitational constant), find the distance of closest approach of the asteroid to the Sun and the speed of the asteroid at the time of closest approach.

3. (**Marks** $= \mathbf{4} + \mathbf{6} = \mathbf{10}$)

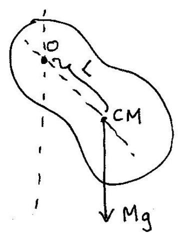
(a) In a one dimensional elastic collision, show that the relative velocity of the two particles after the collision is the negative of the relative velocity before the collision.

In an elastic collision of two particles with masses $m_1$ and $m_2$, the initial velocities are $\mathbf{u_1}$ and $\mathbf{u_2} = \alpha \mathbf{u_1}$. If the initial kinetic energies of the particles are equal, find the conditions on $\frac{u_1}{u_2}$ and $\frac{m_1}{m_2}$ such that $m_1$ is at rest after the collision. Examine both cases for the sign of $\alpha$.

4. (**Marks** $= \mathbf{5} + \mathbf{5} = \mathbf{10}$)

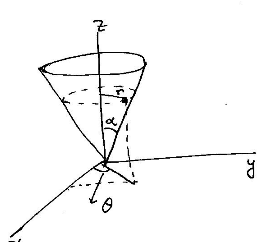



(a) A small ball of radius $r$ and uniform density rolls without slipping near the bottom of a fixed hollow cylinder of radius $R$. What is the frequency of small oscillations ? Assume $r \ll R$




(b) A compound pendulum is a rigid body that oscillates due to its own weight about a horizontal axis that does not pass through the centre of mass of the body (see figure). For small oscillations, show that the period of oscillation is given by $T = 2\pi\sqrt{\frac{k^2}{gL}}$ where the mass of the body is $M$ , the radius of gyration $k$ and $L$ is the distance between the point of suspension and the centre of mass.

5. (**Marks = 3 + 2 + 4 + 1 = 10**)




A particle of mass $m$ is constrained to move on the inside surface of a smooth cone of half-angle $\alpha$ (see figure). The particle is subject to gravitational force.

(a) Write down the Lagrangian of the particle in terms of the generalized coordinates $r$ and $\theta$.

(b) Identify the cyclic coordinate and the corresponding conserved generalized momentum. What symmetry of the Lagrangian corresponds to this conservation law ?

(c) Find the equations of motion. Show that the equation of motion for $\theta$ expresses a conservation law. Which physical quantity is conserved as a consequence of this law ?
(d) Is the Hamiltonian conserved for this system ? Explain.